4.a) Test the convergent of the series

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} + \ldots \infty$$

Soln- Omitting first term,

$$u_n = \frac{(-1)^n}{3^n}$$

$$|u_n| = \frac{1}{3^n} \qquad u_{n+1} = \frac{1}{3^{n+1}}$$

De-Alembart's ratio test-

$$\frac{u_n}{u_{n+1}} = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{1}{3} < 1$$

Divergent

Applying Leibnitz theorem.

Given series is of form $\sum(-1)^n \; u_n = \sum(-1)^n \frac{1}{3^n}$

$$u_n = \frac{1}{3^n}$$

$u_n > 0 \quad n \in N$

$$3^{n+1} > 3^n$$

$$\frac{1}{3^{n+1}} < \frac{1}{3^n}$$

$$u_{n+1} < u_n$$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{3^n} = 0$$

Hence, series is conditionally convergent.