

6.a) Find the maximum and minimum of the function
 $f(x) = x^5 - 3x^4 + 5$

Solⁿ - $f(x) = x^5 - 3x^4 + 5$

$$f'(x) = 5x^4 - 12x^3$$

$$f'(x) = 0$$

$$5x^4 - 12x^3 = 0$$

$$x^3(5x - 12) = 0$$

$$x = 0, x = \frac{12}{5}$$

Critical points are $0, \frac{12}{5}$

$$f''(x) = 20x^3 - 36x^2$$

$$f''(0) = 0$$

$$f''\left(\frac{12}{5}\right) = 20\left(\frac{12}{5}\right)^3 - 36\left(\frac{12}{5}\right)^2 < 0$$

Maxima at point $\frac{12}{5}$.

Minima at point 0

Maximum value of $f(x)$ at $x = \frac{12}{5}$

$$\begin{aligned} f\left(\frac{12}{5}\right) &= \left(\frac{12}{5}\right)^5 - 3\left(\frac{12}{5}\right)^4 + 5 \\ &= \frac{160777}{3125} \end{aligned}$$

Minimum value of $f(x)$ at $x = 0$

$$\begin{aligned} f(0) &= (0)^5 - 3(0)^4 + 5 \\ &= 5 \end{aligned}$$