

b) Discuss the continuity of the function

$$f(x, y) = \begin{cases} x^3 - y^3 & \text{when } x \neq 0, y \neq 0 \\ x^2 + y^2 & \text{when } x = 0, y = 0 \end{cases}$$

Solⁿ - $\lim_{(x, y) \rightarrow (0, 0)} \frac{x^3 - y^3}{x^2 + y^2}$

Consider $y = mx$. Since $(x, y) \rightarrow (0, 0)$
Then $x \rightarrow 0$ at $y = mx$.

$$= \lim_{x \rightarrow 0} \frac{x^3 - m^3 x^3}{x^2 + m^2 x^2}$$

$$= \lim_{x \rightarrow 0} \frac{x - m^3 x}{1 + m^2} = 0$$

$$\text{and } f(0, 0) = 0$$

Hence, function is continuous.