Bihar Engineering University, Patna
End Semester Examination - 2022

Course: B.Tech. *Semester: III* *Time: 03 Hours*
Code: 100312 *Subject: Mathematics-III(PDE, Probability & Statistics)* *Full Marks: 70*

Instructions:-
(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
*(iv) Question No. **1** is compulsory.*

Q.1 ***Choose the correct answer of the following (Any seven question only):*** **[2 x 7 = 14]**

(a) The mean of the Binomial distribution with n observation and probability of success P, is
(i) pq (ii) np (iii) $\sqrt{np}$ (iv) $\sqrt{pq}$

(b) The solution of xp + yq = z is
(i) $f(x^2, y^2) = 0$ (ii) $f(xy, yz) = 0$
(iii) $f(x, y) = 0$ (iv) $f(\frac{x}{y}, \frac{y}{z}) = 0$

(c) Particular integral of $(D^2 - D'^2)\, z = \cos(x + y)$ is
(i) $x \cos(x + y)$ (ii) $\frac{x}{2} \cos(x + y)$
(iii) $x \sin(x + y)$ (iv) $\frac{x}{2} \sin(x + y)$

(d) If the correlation coefficient is 0, the two regression lines are
(i) Parallel (ii) Perpendicular
(iii) Coincident (iv) Inclined at 45^0 to each other

(e) If the mean of exponential distribution

$$f(x) = \begin{cases} Ke^{-kx} & x > 0 \\ 0 & x \le 0 \end{cases}$$

is 1/6 , then the value of K is
(i) 4 (ii) 5 (iii) 6 (iv) 7

(f) Let A and B be any two arbitrary events, then which one of the following is true?
(i) $P(A \cap B) = P(A) \cdot P(B)$ (ii) $P(A \cup B) = P(A) + P(B)$
(iii) $P(A|B) = P(A \cap B)/P(B)$ (iv) $P(A \cup B) < P(A) + P(B)$

(g) Two dice are thrown simultaneously. The probability that at least one of them will have 6 facing up is
(i) 1/36 (ii) 1/3
(iii) 25/36 (iv) 11/36

(h) If a fair coin is tossed four times. What is the probability that two heads and two tails will result?
(i) 3/8 (ii) 1/2
(iii) 5/8 (iv) 3/4

(i) If the mean and variance of a binomial distribution are 5 and 4 respectively, then the value of n is
(i) 10 (ii) 15
(iii) 20 (iv) 25

(j) If the density function of gamma distribution is

$$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{(-x/\beta)}}{\beta^{\alpha} \Gamma\alpha}, & x > 0 \\ 0 & x \le 0 \end{cases}$$

Then mean is equal to
(i) α (ii) β (iii) $\alpha\beta$ (iv) $\alpha\beta^2$

P.T.O.

Q.2 Solve the equations:

(a) $x(y-z)\,p + y(z-x)\,q = z(x-y)$

(b) $\dfrac{\partial^3 z}{\partial^3 x} - 2\dfrac{\partial^3 z}{\partial^2 x\,\partial y} = 2e^{2x} + 3x^2 y$ [7]

Q.3 (a) Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ under the conditions [7]

$u(o,t) = 0, u(l,t) = 0$ *for all* t; $u(x,0) = f(x)$ and $\left(\dfrac{\partial u}{\partial t}\right)_{t=0} = g(x), 0 < x < l.$ [7]

(b) Solve the equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ with boundary conditions

$u(x,0) = 3\sin n\pi x,\ u(0,t) = 0$ *and* $u(1,t) = 0$ *where* $0 < x < 1, t > 0.$ [7]

Q.4 (a) The ends A and B of a rod 20 cm long have the temperature at 30^0C and 80^0C until steady – state prevails. The temperature of the ends are changed to 40^0C and 60^0C respectively. Find the temperature distribution in the rod at time t. [7]

(b) Using the method of separation of variables, solve

$\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, given that $u(0,y) = 8e^{-3y}$ [7]

Q.5 (a) A purse contains 2 silver and 4 copper coins and a second purse contains 4 silver and 4 copper coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin? [7]

(b) Given :

$P(A) = \frac{1}{4}, P(B) = \frac{1}{3}$ *and* $P(A \cup B) = \frac{1}{2}$, *evaluate* $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right), P(A \cap B')$ *and* $P\left(\frac{A}{B'}\right)$. [7]

Q.6 (a) There are three bags: first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. [7]

(b) Fit a poisson distribution to the following:

x =	0	1	2	3	4
f =	46	38	22	9	1

[7]

Q.7 (a) Find Pearson's coefficient of skewness for the following data: [7]

Class :	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Frequency :	5	9	14	20	25	15	8	4

(b) A set of five similar coins is tossed 320 times and the result is [7]

No. of heads	: 0	1	2	3	4	5
Frequency	: 6	27	72	112	71	22

Test the hypothesis that the data follow a binomial distribution.

Q.8 Let the joint probability density function of the continuous random variables x and y be [14]

$$f(x,y) = \begin{cases} kxy; & 0 < x < 2,\ 1 < y < 3 \\ 0; & \textit{elsewhere} \end{cases}$$

Find the value of K and probability density function of x + y. Also find the mean and variance of x and y

Q.9 (a) Prove that: [7]

$$(1 - x^2)\,P_n'(x) = n\,[P_{n-1}(x) - x\,P_n(x)]$$

Where $P_n(x)$ is the legender's polynomial of the first kind.

(b) Prove that : [7]

$$\frac{d}{dr}[x^n J_n(x)] = x^n J_{n-1}(x)$$